An Introduction to Fully Coupled Acoustical Systems including Dissipation Mechanisms

D. Giljohann, Bad Soden and A. Landfester, TU Darmstadt

When analyzing vibrating structures the influence of the medium in the surroundings of the structure is often neglected. In most cases this approach is justifiable, particularly if the emitting sound source is small in size compared with the wavelength and is stiff in relation to the medium [1]. In recent years the trend to light-weight structures for aeronautical and transportation use like trains, busses, ships, planes or rocket components often leads to the following question: Is there a frequency range where the presence of the exterior domain has a significant influence on the power radiated by the structure or concerning psychological effects of acoustical perception? Software producers of numerical codes in the field of finite element or boundary element methods recognizing the requirements of their customers are developing software components for the numerical treatment of fully coupled structural vibration and acoustic radiation. As example problems often cylindrical structures are selected since air and marine vehicles are often composed of cylindrical sections and most of the standard literature covers these types of structures. However, for the beginner in coupled acoustic analysis it may be easier to consider a vibrating sphere of order zero which means that the sphere performs purely radial displacements. The analytical expressions for the structural vibration and the radiation of sound by this type of sphere derived in this paper consist only of simple functions. This makes the problem treatable for our discussion of coupled acoustical problems although it is of low technical importance since there do not appear any bending waves on the surface of the structure.

Equations of structural vibration

The differential equation of a vibrating sphere of order zero is given by [2] as

$$\frac{2\underline{E}h}{R^2(1-\nu)}w(t) + \rho_\mathrm{s} h \frac{\partial^2 w(t)}{\partial t^2} = p_\mathrm{i}(t) - p_\mathrm{e}(t) \tag{0.1}$$

where the index e denotes the acoustic properties of the medium in the exterior of the sphere and the index s the structural quantities, density ρ and Poisson number ν. η_s is the structural loss factor of the complex elastic moduli $\underline{E}$

$$\underline{E} = E(1 + i\eta_\mathrm{s}). \tag{0.2}$$

The geometrical quantities h and R are marked in fig. 1. The radial displacements $w(t)$ of the sphere are driven by a known internal pressure loading p_i. A medium in the interior of the sphere is not considered. The pressure $p_\mathrm{e}(t)$ in the exterior domain at radius R is a priori unknown. Assuming time harmonic excitation of frequency Ω we get

$$\frac{2E(1+i\eta_\mathrm{s})h}{R^2(1-\nu)}w - \Omega^2 \rho_\mathrm{s} h w = p_\mathrm{i} - p_\mathrm{e}. \tag{0.3}$$

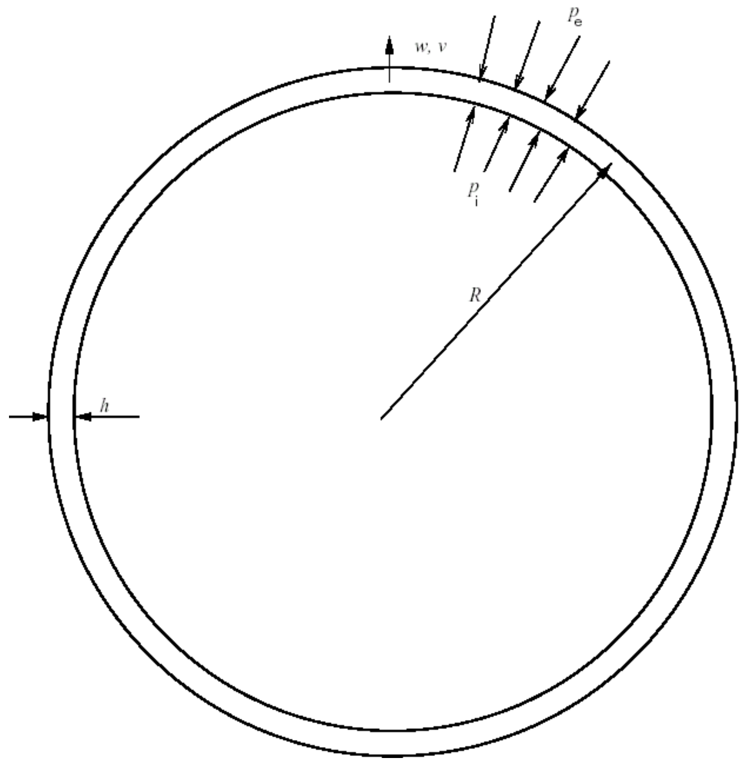

Abb. 1: Plane cut through midplane of sphere including acoustic quantities.

From eq. (0.3) the undamped eigenkreisfrequency ω is calculated as

$$\omega = \sqrt{\frac{2E}{R^2 \rho_{\rm s}(1-\nu)}} \tag{0.4}$$

from the homogeneous part of the differential equation. At absence of an exterior fluid ($p_{\rm e} = 0$) we get in-vacuo vibrations at the kreisfrequency $\Omega := \kappa\omega$ with the admittance

$$\frac{v(\Omega)}{p_{\rm i}} = \frac{\kappa\omega R^2 \rho_{\rm e} c_{\rm e}}{2Eh/(1-\nu)[i(\kappa^2-1)+\eta_{\rm s}]} \tag{0.5}$$

and κ being the ratio of radial eigenfrequency. For the velocity in normal direction $v(\Omega)$ of eq. (0.5) the relationship $v = i\Omega w$ is used.

Basics of acoustical coupling

The exterior pressure can be understood as a reaction force which is created by the acoustic radiation of the sphere. It can be considered as a function of the in-vacuo modes of the structure [1]. Assuming linear behaviour of the structure including small oszillations of the radiating surface Γ, $p_{\rm e}(\boldsymbol{x})$ can be expressed in terms of the structural eigenfunctions $\varphi_i(\boldsymbol{x})$ by

$$p_\mathrm{e}(\Omega,\boldsymbol{x}) = \sum_i A_i(\Omega)\boldsymbol{\varphi}_i(\boldsymbol{x})\cdot\boldsymbol{n}(\boldsymbol{x}) \qquad i=1,2,\dots \quad \forall\,\boldsymbol{x}\in\Gamma \tag{0.6}$$

The unit normal vector $\boldsymbol{n}(\boldsymbol{x})$ on the surface assures that only the normal part of the eigenfunctions is taken into account because only the normal component affects the radiation. In our case the eigenfunctions $\varphi_i(\boldsymbol{x})$ are given as the eigenvectors of the homogeneous equation (0.3). Due to the simplicity of this problem only one eigenfunction $\varphi_i(\boldsymbol{x}) = 1$ has to be considered here. The unknown coefficient $A_1(\Omega) = p_\mathrm{e}(\Omega)$ can be calculated using the impedance Z of a vibrating sphere of order zero

$$Z := \frac{p_\mathrm{e}}{v} = \frac{p_\mathrm{e}}{i\Omega w} = \rho_\mathrm{e}c_\mathrm{e}\frac{\Omega R + i\eta_\mathrm{e}\Omega R + ic_\mathrm{e}}{(\Omega R)^2 + 2\eta_\mathrm{e}c_\mathrm{e}\Omega R + c_\mathrm{e}^2} \tag{0.7}$$

with η_e being the loss factor of the external medium which is introduced by a complex wavenumber

$$\underline{k} := k(1 - i\eta_\mathrm{e}). \tag{0.8}$$

Then the unknown coefficient is acchieved by

$$A_1(\Omega) = i\Omega Z. \tag{0.9}$$

The expression of eq. (0.3) in v using the impedance relation (0.7) yields for the dimensionless admittance

$$\rho_\mathrm{e}c_\mathrm{e}\frac{v(\Omega)}{p_\mathrm{i}} = \frac{\kappa\omega R^2\rho_\mathrm{e}c_\mathrm{e}}{2Eh/(1-\nu)[i(\kappa^2-1)+\eta_\mathrm{s}] + ZR^2\kappa\omega}. \tag{0.10}$$

The dissipation mechanisms in structure and exterior fluid have their most significant influence on the admittance at the eigenfrequency ω. By the use of eq. (0.4) one gets

$$\rho_\mathrm{e}c_\mathrm{e}\frac{v(\omega)}{p_\mathrm{i}} = \frac{\omega R^2\rho_\mathrm{e}c_\mathrm{e}}{\eta_\mathrm{s}2Eh/[1-\nu] + ZR^2\omega}. \tag{0.11}$$

At absence of an exterior fluid ($\rho_\mathrm{e} = 0$) it is further simplified to

$$\frac{v(\omega)}{p_\mathrm{i}} = \frac{\sqrt{1-\nu}}{\eta_\mathrm{s}\sqrt{2\rho_\mathrm{s}Eh}}. \tag{0.12}$$

Damping mechanisms in structure and fluid

Simulation experts are recommended to get the damping informations from measurements since the joints between the different parts of an assembly are influencing the damping more than the pure material damping. Both damping coefficients are combined to the structural damping factor η_s. In tab. 1 typical values for the structural damping are given [3].

Fluid damping is not commonly used although it has a significant influence on the

Pure material damping metals	Structural damping
$10^{-4} \le \eta_s \le 2 \cdot 10^{-3}$	$10^{-2} \le \eta_s \le 5 \cdot 10^{-2}$

Table 1: Loss factors η_s for material and structural damping

sound level in large scale acoustics. For example the loss of sound power due to fluid damping should not be neglected in the case of traffic noise like that of a landing plane in the vicinity of an airport. From eq. (0.8) the complex speed of sound can be interpreted as an appropriate material parameter of the fluid

$$\underline{k} = \frac{2\pi f}{\underline{c}}, \qquad \underline{c} = \frac{c}{1 - i\eta_e}. \tag{0.13}$$

The loss factor η_e depends on the kreisfrequency Ω and on the temperature. For different thermal conditions of air the values of η_e are plotted in fig. 2 [4], [5]. The loss factor η_e is highly effected by the relative humidity of the air.

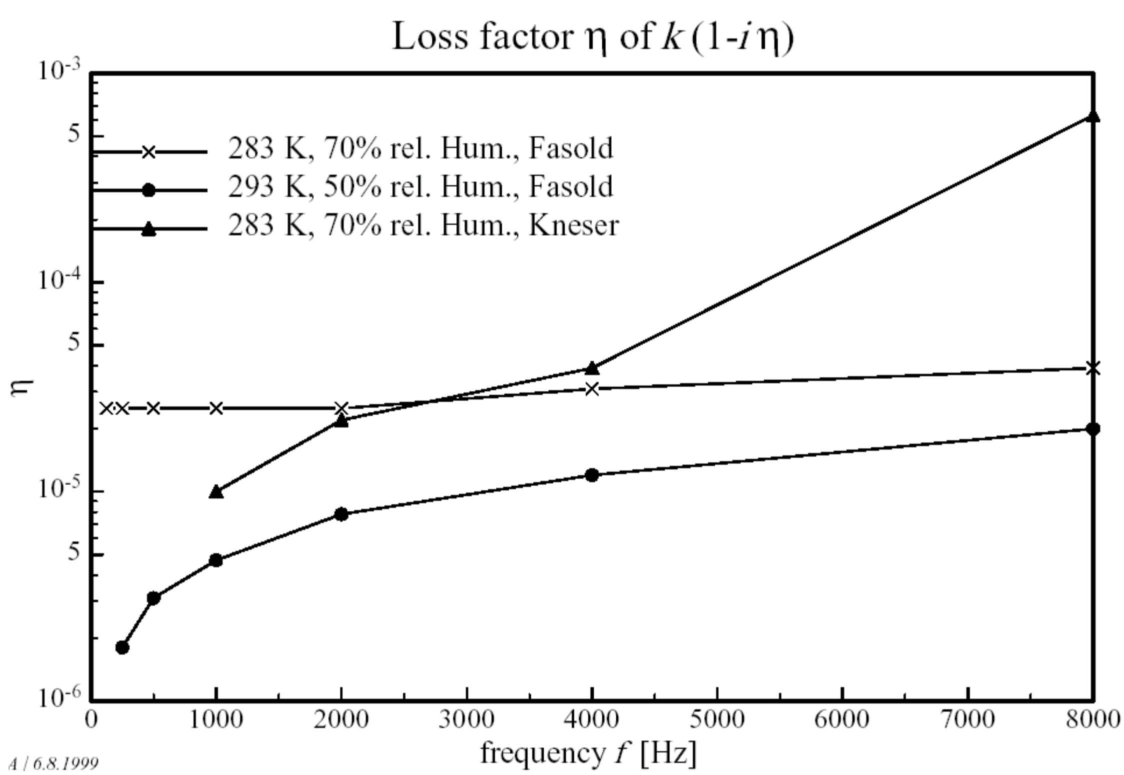

Abb. 2: Loss factor of frequency $f = \Omega/(2\pi)$ in air.

Results and Discussion

In fig. 3 and 4 the dimensionless admittances of eq. (0.10) are plotted for the vibration of the sphere in air and water. Both media are of high technical importance and are quite different concerning their fluid properties:

$$\rho_{\text{air}} = 1.20\,\text{kg/m}^3, \qquad c_{\text{air}} = 340\,\text{m/s}$$

$$\rho_{\text{water}} = 998\,\text{kg/m}^3, \qquad c_{\text{water}} = 1500\,\text{m/s}. \tag{0.14}$$

For both fluids the same fluid loss factor was used: At the frequency of 4000 Hz the representative value of $\eta_e = 3 \cdot 10^{-5}$ was chosen.
It is shown in both figures that the proper modelling of the structural damping can be of more influence regarding the admittance than the coupling to the fluid.

Finally the transition to a different medium is regarded. Neglecting all dissipative effects for the ratio of velocities when changing the fluid from water to air one gets

$$\frac{v_{\text{air}}}{v_{\text{water}}} = \frac{Z_{\text{water}}}{Z_{\text{air}}} = \frac{\rho_{\text{water}} c_{\text{water}} \cdot (\omega R - i c_{\text{air}})}{\rho_{\text{air}} c_{\text{air}} \cdot (\omega R - i c_{\text{water}})} \approx 3600. \tag{0.15}$$

Eq. (0.15) is the maximum value at the structural eigenfrequency. In fig. 5 the ratio of velocities is plotted. In our case of structural in plane waves the fluid only affects the structural vibrations at the eigenfrequency. The interaction between bending waves and sound waves in the fluid are more complicated and may be quite different. Analytical solutions in those cases contain cylindrical and spherical functions. So the vibrating sphere of order zero is well suited as an appropriate test case in the upcoming field of numerical codes concerning fully coupled acoustic analysis.

References

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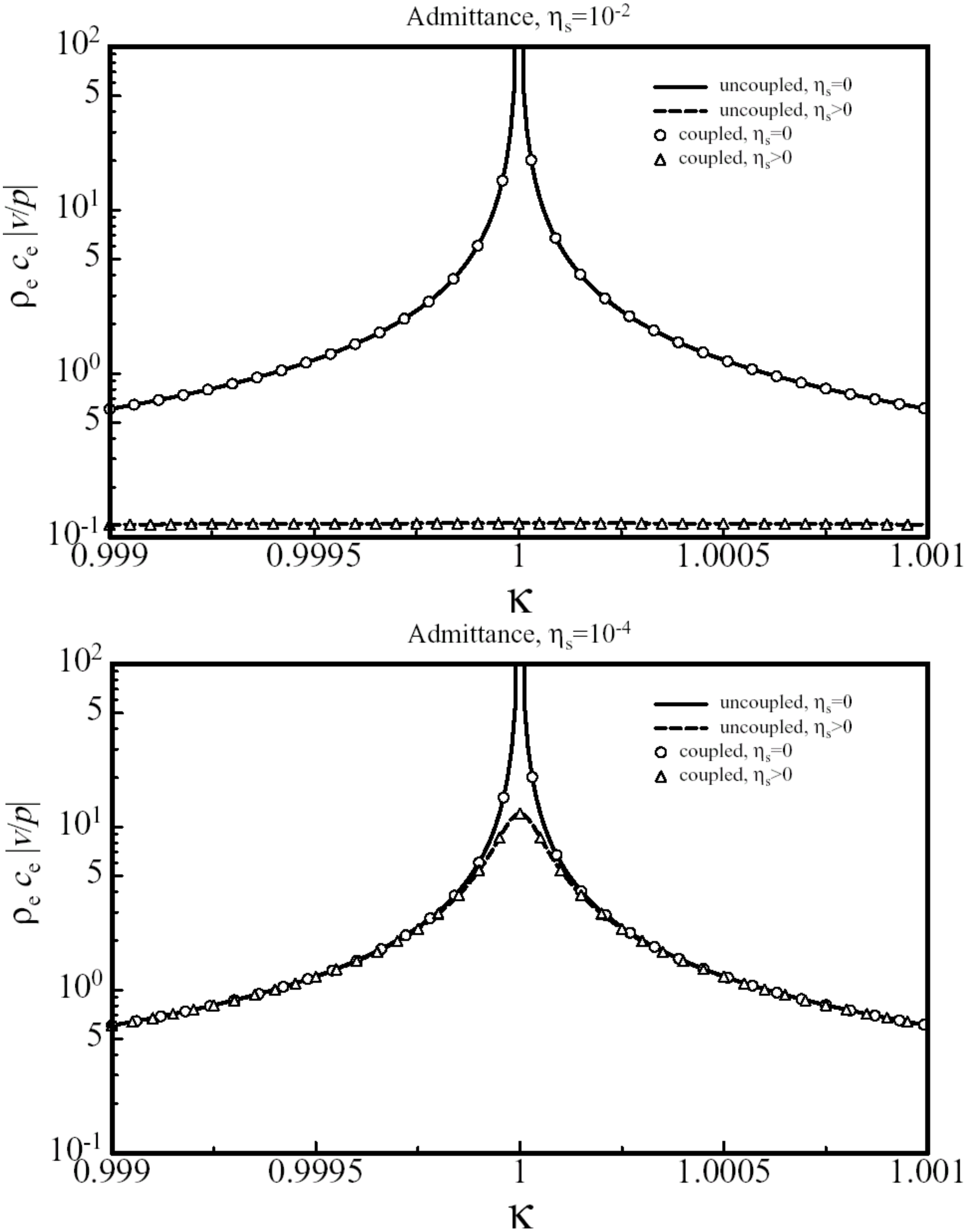

Abb. 3: Dimensionless admittance $\rho_\mathrm{e} c_\mathrm{e} |v(\omega)/p_\mathrm{i}|$ of sphere in air $\rho_\mathrm{e}/\rho_\mathrm{s} = 1.5 \cdot 10^{-4}$, variation of the structural loss factor.

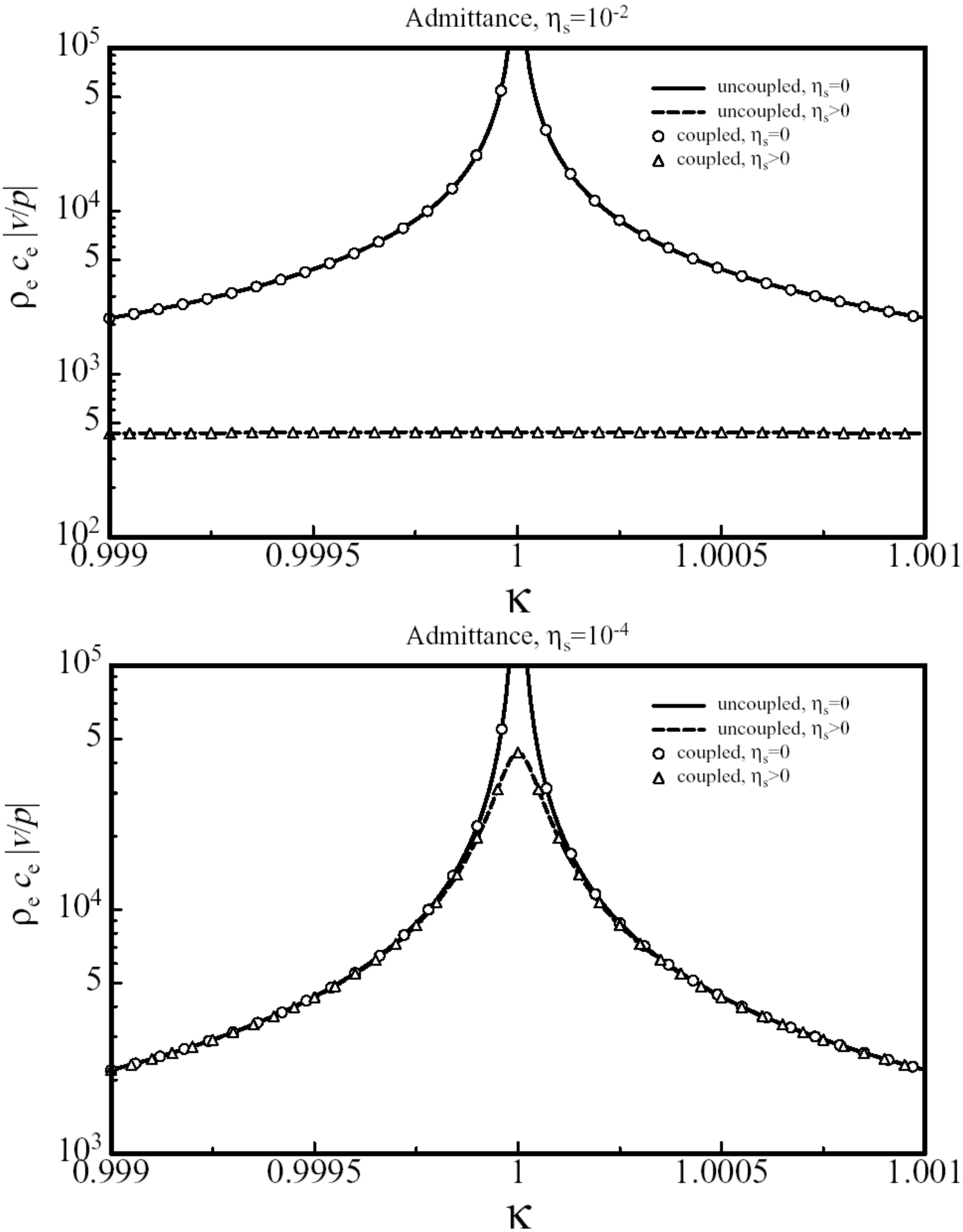

Abb. 4: Dimensionless admittance $\rho_e c_e |v(\omega)/p_i|$ of sphere in water $\rho_e/\rho_s = 0.127$, variation of the structural loss factor.

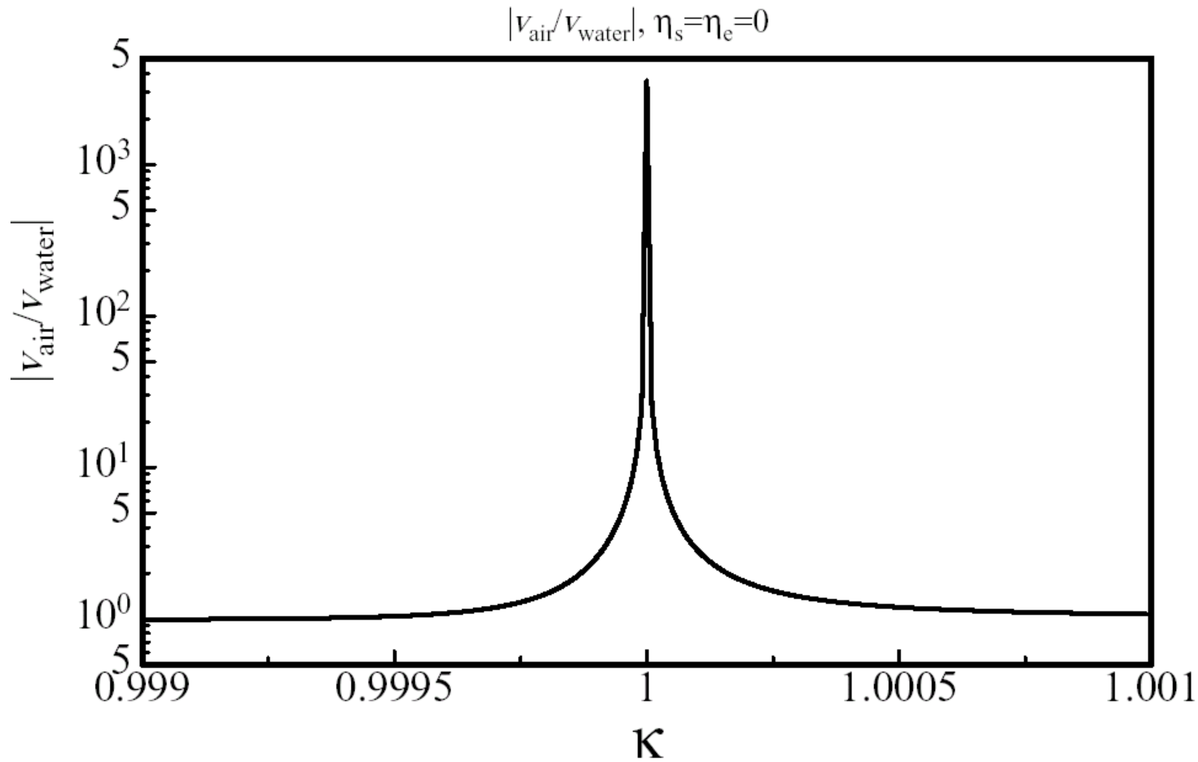

Abb. 5: Ratio of velocities in different fluids.