Finite/Infinite Element Methods for the Helmholtz Equation in large Exterior Domains including Damping and Flow

by

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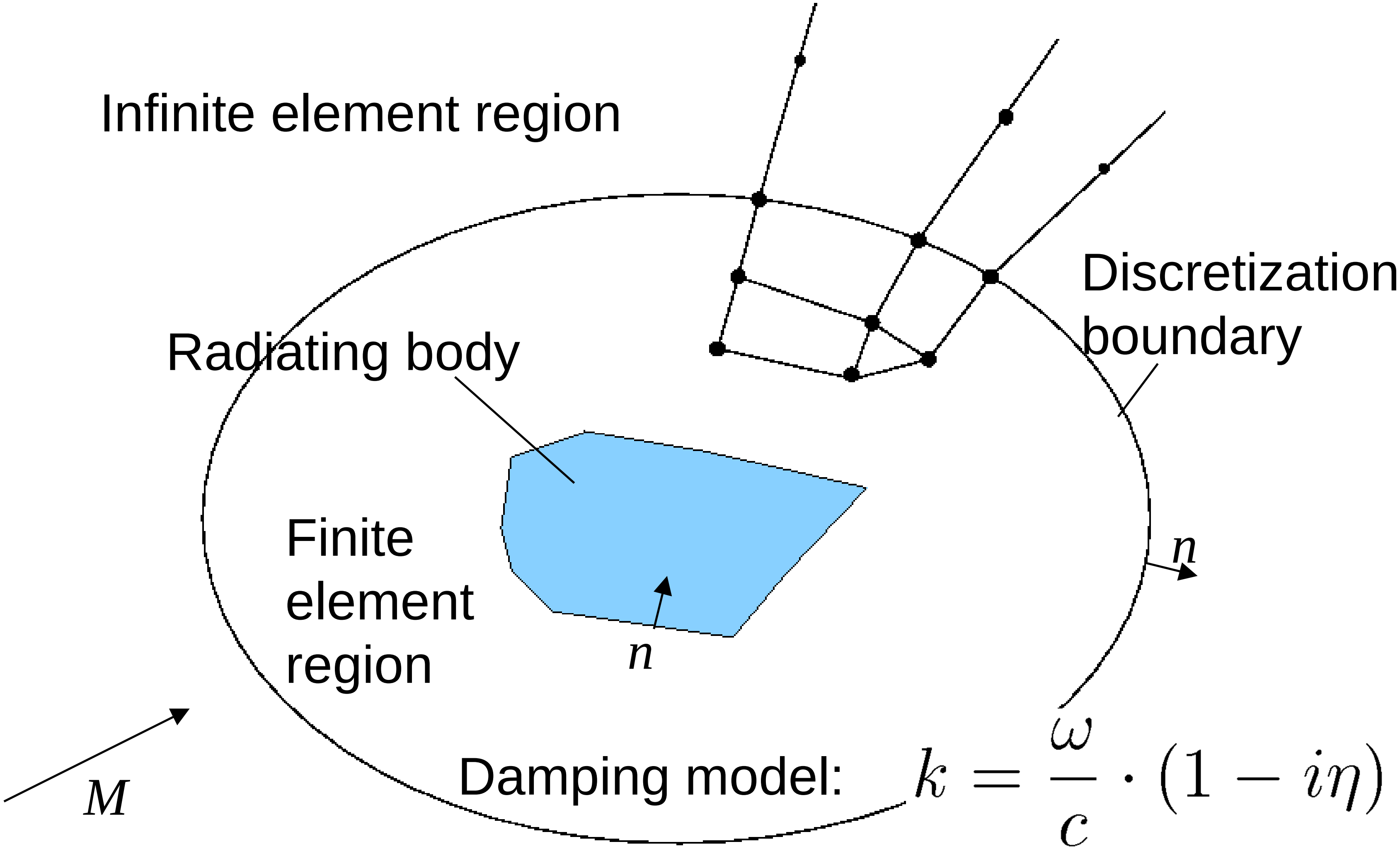
ICSV 7, Garmisch 2000

Problem of interest



Damping model: $k = \frac{\omega}{c} \cdot (1 - i\eta)$

Basic equations (physics)

Helmholtz equation:

$$\Delta\Phi + k^2\Phi = q$$

Sommerfeld radiation condition:

$$\lim_{r\to\infty} r\left(\frac{\partial\Phi}{\partial r} - ik\Phi\right) = 0$$

Boundary condition:

$$\nabla\Phi \cdot \boldsymbol{n} = -v_{\mathrm{n}}\left(\boldsymbol{x}\right)$$

Transformations

Assumptions:

- large distance between observer and radiating body
- uniform, undisturbed flow (moving media principle)
- potential related to the maximum volume flux on the surface of the body

Subsonic case:

$$\Phi(\boldsymbol{x}, k, M < 1) = \frac{\Phi(\boldsymbol{x}, k, M = 0)}{\sqrt{1 - M^2 \sin^2 \alpha}} \, e^{ikr\left(1 + \frac{M \cos\alpha - \sqrt{1 - M^2 \sin^2 \alpha}}{M^2 - 1}\right)}$$

Supersonic case:

$$\Phi(\boldsymbol{x}, k, M > 1) = \begin{cases} \frac{\Phi(\boldsymbol{x}, k, M=0)}{\sqrt{1 - M^2 \sin^2 \alpha}} \, e^{ikr\left(1 + \frac{2M \cos\alpha}{M^2 - 1}\right)}, & |\alpha| < \arcsin(\frac{1}{M}) \\ 0 & |\alpha| \geq \arcsin(\frac{1}{M}) \end{cases}$$

Basic equations (FEM/IFEM)

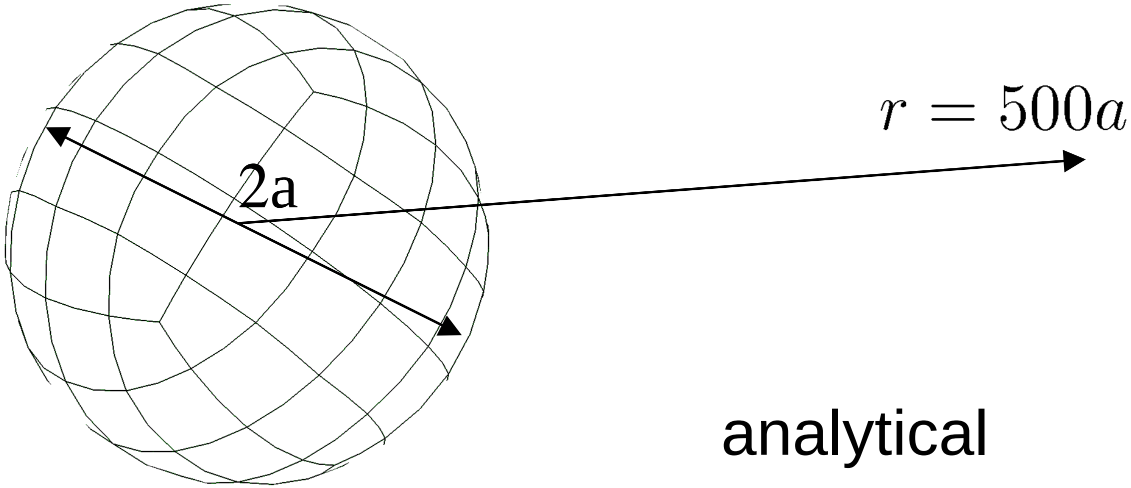
- Spherical and ellipsoidal coordinates (Burnett & Holford)
- Trial functions of different order m:

$$\Phi\left(R,\theta,\phi,k\right) \sim e^{-ikR} \sum_{n=1}^{m} \frac{F_n\left(\theta,\phi,k\right)}{R^n}$$

- Test functions with conjugated and unconjugated formulations (Astley et al.):

$$w_j \sim \begin{cases} e^{ikR} \sum_{n=1}^{m} \frac{F_n(\theta,\phi,k)}{R^{n+2}} \\ e^{-ikR} \sum_{n=1}^{m} \frac{F_n(\theta,\phi,k)}{R^n} \end{cases}$$

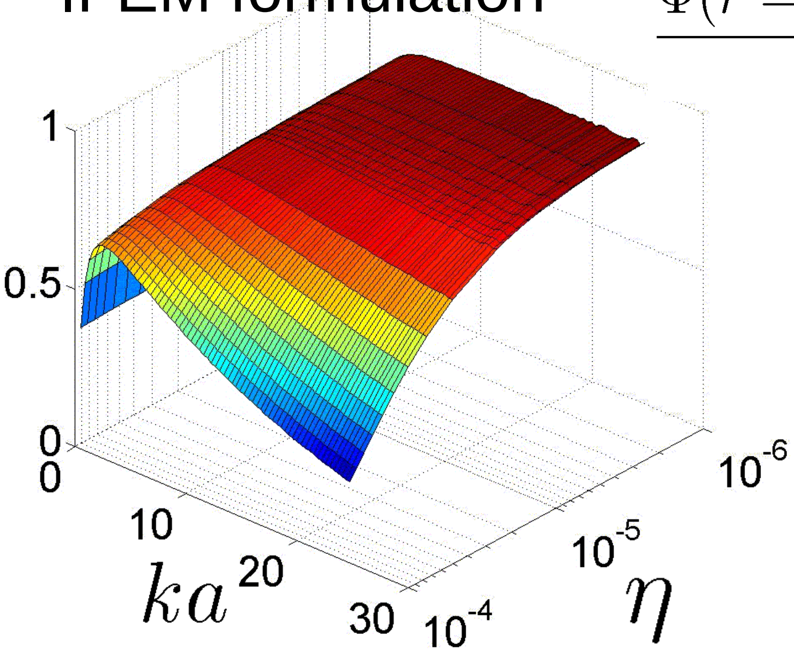
Potential of sphere, order 0, *r*=500*a*, *M*=0

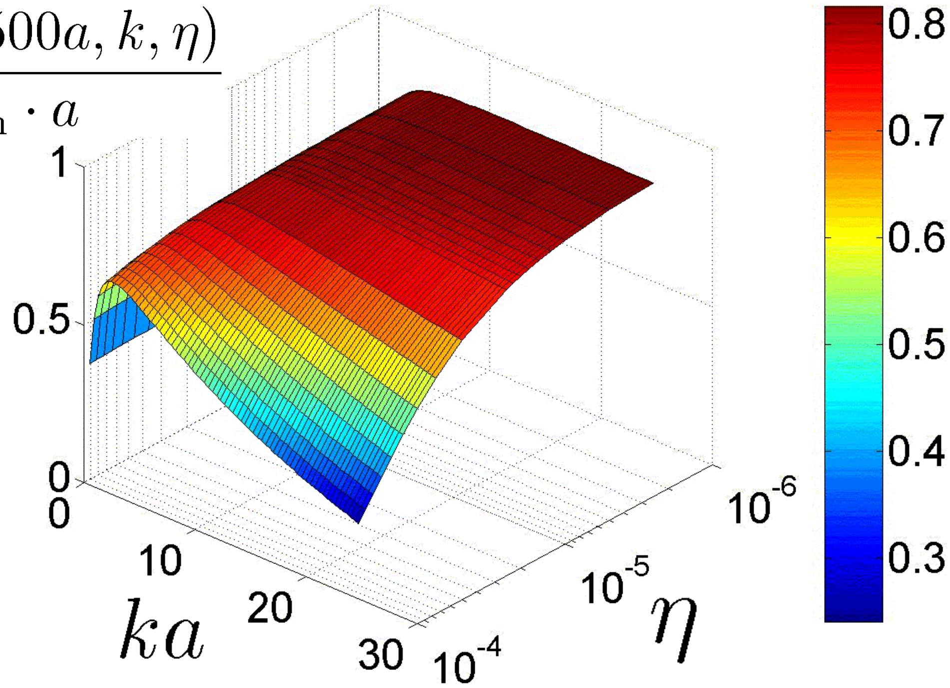


Spherical coord., conjugated IFEM formulation

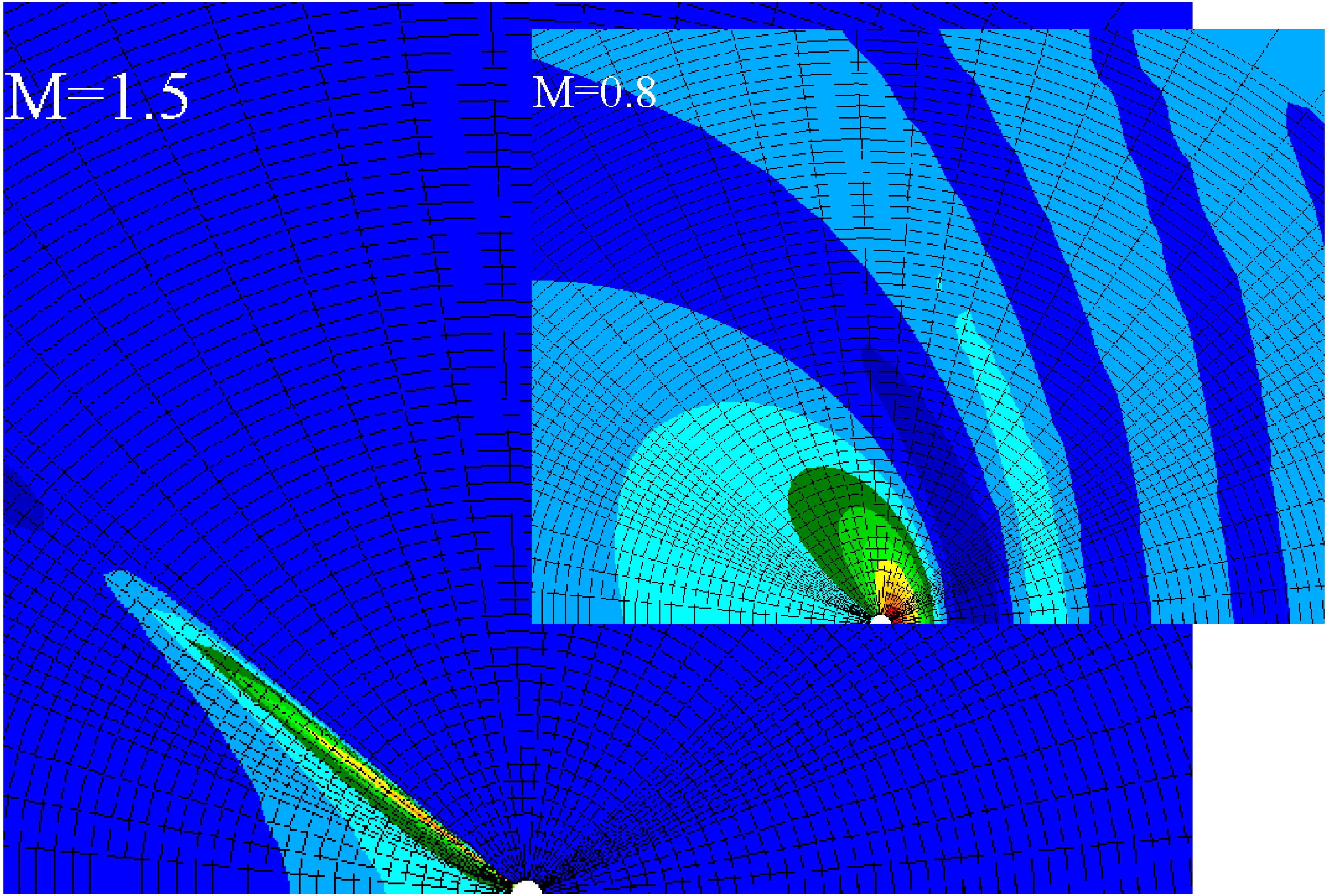
analytical

$$\frac{\Phi(r = 500a, k, \eta)}{v_{\mathrm{n}} \cdot a}$$





Sphere, order 0 in flow



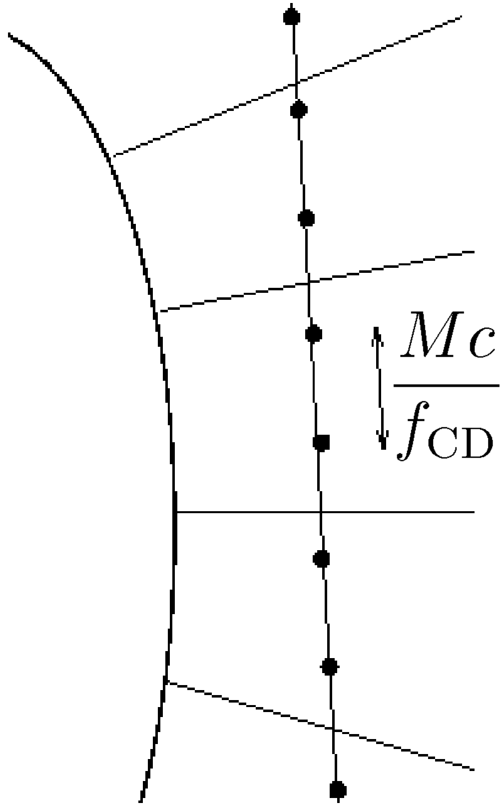
Sound Samples (sphere of order 0, analytical)

1000 Hz, M=0.2

1000 Hz, M=0.5

2000 Hz, M=0.2

2000 Hz, M=0.5



1000 Hz, M=1.5 limitations of our physical model and the CD recording quality

Summary & future research

- Simple flow model for acoustic radiation with damping properties
- no extra computational costs
- well suited for infinite element formulations
- accurate results

- infinite element postprocessing for acoustic perception